Mahalanobis Distance: A Multivariate Measure of Effect in Hypnosis Research

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Mahalanobis distance, a multivariate measure of effect, can improve hypnosis research. When two groups of research participants are measured on two or more dependent variables, Mahalanobis distance can provide a multivariate measure of effect. Finally, Mahalanobis distance is the multivariate squared generalization of the univariate d effect size, and like other multivariate statistics, it can take into account the intercorrelation of variables. **(Sleep and Hypnosis 2007;9(2):67-70)**

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EFFECT SIZES

Effect sizes allow researchers to determine if statistical results have practical significance, and they allow one to determine the degree of effect a treatment has within a population; or simply stated, the degree in which the null hypothesis may be false (1-3).

THE EFFECT SIZE D

Cohen (1977) defined the most basic effect measure, the statistic that is synthesized, as an analog to the t-tests for means. Specifically, for a two-group situation, he defined the d effect size as the differences between two population means divided by the standard deviation of either population, because homogeneity or equality of variances is assumed. Even though it may not be apparent, the d effect size is a distance measure. For example, it is the distance or difference between population means expressed in standard deviation units. Equation (1) has the general formula:

(1) $$d = \frac{\mu_1 - \mu_2}{\sigma}$$

μ_1 = the treatment group

μ_2 = the control group

σ = population standard deviation

Suppose μ_1 equals the population mean, and in this case we are using it to represent the treatment group population mean of 1.0. And let us assume that μ_2, the population mean for the control group, equals .2, and, finally, σ = 1.00. By substitution,

$$d = \frac{1.0 - .2}{1} = .8$$

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Now, this formula is simple, and where do the problems begin? First the σ or population standard deviation can be from the control group posttest measure; it can be the pretest standard deviation for the control group, and it can be the pooled or weighted standard deviation that involves both groups. Therefore, within a study, at least three different d effect sizes can be obtained. First, one based on the control group posttest measure standard deviation, second, another based on the control group pretest measure, and a d effect size measure based on the average standard deviation for the treatment and control group. The d effect sizes from several studies can be averaged, and the result is an overall effect for a series of studies.

Unlike statistical significance testing, which attempts to reject or to fail to reject the null hypothesis (the population means are equal) meta-analysis and its effect size measures address practical significance, or the degree to which the null hypothesis may be false.

DEFINITIONS OF MULTIVARIATE STATISTICS

The term multivariate can be a confusing term, but in one sense it involves examining several variables simultaneously. Within a regression context, it is the relationship between two or more predictors (independent variables) and a dependent variable. From a multivariate regression context, it involves the relationship between two or more predictors and two or more dependent variables. Within the correlational multivariate methods, these include path analysis, factor analysis, principal components analysis, canonical correlation, and predictive discriminant analysis.

When two or more groups of participants are measured on several dependent variables, this is a multivariate analysis of variance (MANOVA), a multivariate extension of ANOVA. Other examples of multivariate statistics are multivariate analysis of covariance (MANCOVA), a multivariate generalization of ANCOVA, step down analysis, a multivariate test procedure that focuses on the ordering of dependent variables through a series of analyses of covariances, descriptive discriminant analysis, a multivariate technique that determines group membership, and log linear analysis, an extension of the chi-square test to three or more variables. In summary, if participants are measured on two or more dependent variables, a multivariate situation exists; however, multivariate statistics are complex and can be confusing.

Why are multivariate statistics important? First, they control type I error, but with many univariate tests it cannot be easily estimated. Second, univariate statistics do not take into account the correlations among variables. Finally, multivariate statistics are more powerful than univariate statistics.

Hotelling's T^2

Hotelling's T^2 is the squared multivariate generalization of the t test. The univariate t test is the following:

$$(2)\quad t = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Hoteling's T^2 is the following:

$$(3)\quad T^2 = \frac{n_1 n_2}{n_1+n_2}\left(\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2\right)' \mathbf{S}^{-1}\left(\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2\right)$$

bold letters indicate a matrix

S- sample covariance matrix

S^{-1} matrix analogue of division is inversion

$\left(\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2\right)$ transpose of vectors of means

$(\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2)$ vector of means

The connection between Hotelling's T^2 and F is the following:

(4) $F = \frac{n_1 + n_2 - p - 1}{(n_1 + n_2 - 2)p} T^2$

This formula shows that T^2 can be transformed into a F statistic.
n_1 is the sample size of group one
n_2 is the sample size of group two
p is the number of dependent variables

The univariate d and Mahalanobis distance (D^2) are the following:
univariate multivariate

(5) $d = \frac{\bar{y}_1 - \bar{y}_2}{s}$ (6) $D^2 = (\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2)' \mathbf{S}^{-1} (\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2)$

With equations 5 and 6, one can see that the univariate and multivariate statistics follow the same form.
D^2 is also the following formulas. Formula 8 shows how the intercorrelation of variables is taken into account:

(7) $[(n_1 + n_2)/n_1 n_2]T^2$

(8) $\frac{1}{1-r^2}\left[\frac{(x_{i1} - \bar{x}_1)}{s_1^2} + \frac{(x_{i2} - \bar{x}_2)}{s_2^2} - \frac{2r(x_{i1} - \bar{x}_1)(x_{i2} - \bar{x}_2)}{s_1 s_2}\right]$

And T^2 is the following:

(9) $[n_1 n_2 / (n_1 + n_2)]D^2$

Let us consider a two-group problem for Hotelling's T^2. Suppose two forms of treatment – Person-Centered and Rational Emotive Behavior Therapy (REBT) hypnosis – are used to reduce state anxiety and trait anxiety in college students. There are three participants in the Person-Centered group and six in the REBT hypnosis group. State and trait anxiety has a mean of 50 and a standard deviation of 10. The following are these fictitious data:

Person-Centered		REBT	
State Anxiety	Trait Anxiety	State Anxiety	Trait Anxiety
51	53	54	56
53	57	56	58
52	52	56	58
		55	60
		55	60
		54	66

The following are the SPSS codes for these data:

```
Title 'Two-Group Manova'.
Data list free/gp dep1 dep2.
Begin data
1 51 53
1 53 57
1 52 52
2 54 56
2 56 58
2 56 58
2 55 60
2 55 60
2 54 66
End data.
Manova dep1 dep2 by gp(1,2)/
  Print=cellinfo(means)/.
```

The following selected output from SPSS has the results of Hotelling's T^2:

EFFECT .. gp
Multivariate tests of Significance (S = 1, M = 0, N = 2)

Test Name	Value	Exact F. Hypoth.	DF	Error DF	Sig. of F
Pillais	.80855	12.66972	2.00	6.00	.007
Hotellings	4.22324	12.66972	2.00	6.00	.007
Wilks	.19145	12.66972	2.00	6.00	.007
Roys	.80855				

Note. . F statistics are exact.

EFFECT . . gp (Cont.)
Univariate F-tests with (1,7) D. F.

Variable	Hypoth. SS	Error SS	Hypoth. MS	Error MS	F	Sig. of F
dep1	18.00000	6.00000	18.00000	.85714	21.00000	.003
dep2	64.22222	73.33333	64.22222	10.47619	6.13030	.042

$D^2 = N (T^2) / n_1 n_2 = 9 (4.22)/18 = 2.11$, and this is a large effect size.

Unlike univariate statistics, Mahalanobis distance takes into account the intercorrelation of variables. D^2 is known within a regression context to locate outliers on predictors, and a large value suggests an outlier on a predictor.

Stevens (5) provided power values for D^2, and he found for values of $D^2 <.64$ and $n < 25$, power is usually poor (<.45) and seldom adequate >.70. Usually hypnosis researchers will report the multivariate T^2, and one can calculate D^2 using equation 7. In summary, Mahalanobis distance, or D^2, is the multivariate analogue of the univariate d effect size. Stevens (6) provided the following guidelines for interpreting D^2:
$D^2 = .25$ (small effect)
$D^2 = .5$ (medium effect)
$D^2 = >1$ (large effect)

DISCUSSION

Mahalanobis distance (D^2) is a multivariate measure of effect, and it is a multivariate measure of distance. For example, large values of D^2 indicate large effect sizes. To summarize:

(10) $$D_1^2 = (\mathbf{X}_1 - \overline{\mathbf{X}})' \mathbf{S}^{-1} (\mathbf{X}_1 - \overline{\mathbf{X}})$$

D^2 is defined in terms of the covariance matrix of S. X_i equals the vector of the data for case i, and $\overline{X}$ is the vector of means, or as statisticians call the centroid for predictors within the regression context. D^2 is essential for improving hypnosis research, and it can be helpful in locating outliers and influential data points in regression analysis (Stevens, 1984). Confidence intervals for D^2 is an area for additional research. For example, Hess, Hogarty, Ferron, and Kromney (7) found that confidence intervals for D^2 were large and uninformative, and they presented SAS/IML code employed in a macro called D2BAND and for these intervals. Finally, Mahalanobis distance is a multivariate measure of effect, and it can improve hypnosis research.

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